Comparison of Single-Layer and Multi-Layer Windings with Physical Constraints or Strong Harmonics

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Found in *IEEE International Symposium on Industrial Electronics,* July 2006, pp. 1467–1473.

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***Abstract*— Single-layer and multi-layer transformer and inductor windings are compared. Multi-layer windings (including litz wire) can typically achieve lower loss, but several situations that may favor single-layer windings are discussed. As shown in previous work, for sinusoidal waveforms, if the minimum practical layer thickness is 1.5 skin depths or greater, single-layer windings are preferred. If the maximum number of practical layers is limited, increasing the number of layers decreases the loss approximately inversely proportional to the square root of the number of layers. For waveforms with strong harmonic content, it becomes more difficult to reduce losses with multi-layer windings. We study typical waveforms for power converter circuits and conclude that, for waveforms used in power electronics, multi-layer windings have significant advantages even with only a few layers. The number of layers needed can be as few as two layers for many waveforms; other waveforms, such as square waveforms with fast rise times, typically require four to six layers to effect a significant improvement over a single-layer design.**

I. Introduction

Since the 1920s the technique of using multiple layers in a winding design to reduce high-frequency loss has been well known [1], [2], [3], [4], [5]. While having layers thin compared to a skin depth is beneficial, having many layers can increase loss due to proximity effects. Thus, this approach can actually lead to increased losses if the winding is poorly designed. Although the eddy-current losses can be small, they increase in proportion to frequency squared. This effect is well understood and addressed in the literature, and methods for optimizing multi-layer windings have been developed to avoid incurring excessive loss. For example, optimization of foil windings is addressed in [6], optimization of solid-wire winding in [5], and optimization of litz-wire windings in [7], [8], [9].

In some cases, however, a single-layer winding may be better than any multi-layer winding. In a single-layer winding with high-frequency current, the current flows on the surface in a layer one skin depth deep. Because there are no other layers inducing a field, the loss depends only on the resistance of the layer where current flows, and is proportional to the square root of frequency. In the Appendix of [10] it is shown that the performance of any given multi-layer winding degrades at high frequency and eventually becomes worse than that of a single-layer winding. However, one cannot conclude that a single-layer winding is therefore in general superior for high frequencies, because, for any given high frequency, making the layers thinner in the multi-layer winding can reduce high-frequency losses arbitrarily.

But there are four arguments in favor of single-layer windings that cannot be dismissed so readily [10]. Firstly, very high frequencies may require thinner conductor layers or thinner wire than is practical. Secondly, the additional effort required for a multi-layer design may be impractical in comparison to the achievable decrease in loss, particularly if a large number of layers is required. Thirdly, the current waveform may not be a single frequency: it may contain harmonics. Even if the design is optimized for low fundamental-frequency resistance, the harmonics may incur substantial loss if the loss increases in proportion to frequency squared. And finally, the current waveform may contain a large dc component. Even if the design is optimized for low ac resistance, the dc losses may be larger than the ac losses.

Thus, in many situations, it is not immediately clear whether a multi-layer or single-layer winding is the best strategy. Our goal in this work is to provide guidance for designers on which strategy is preferred for any given design. Models for analyzing loss in all of the situations discussed above are well established. (See, for example, [11] and [12] for discussion and experimental validation of recent modeling improvements.) The contribution of this paper is not to improve modeling methods, but rather to provide designers with strategies for selecting types of windings that may be most effective in particular types of designs. We make use of modeling methods that are already well established.

Some of these arguments in favor of single-layer windings were considered in [10]. This paper reviews the analysis in [10] for cases in which the layer thickness is constrained in Section III. In Section IV we add analysis of cases in which the number of layers is constrained. In Section V we correct and extend the work on waveforms with significant harmonic content in [10], and present new results showing that multilayer windings can be advantageous for waveforms with strong harmonic content, even with just a moderate number of layers. These results are quantified for different waveforms. Another issue that is described above, but is not addressed in this paper, is windings with significant dc current. That situation is analyzed in [13].

In order to make it easy for designers to use our results, Section VI presents simple practical formulas and rules for design. We begin with a review of the problem to be addressed and of the loss models used in Section II.

II. Problem Definition and Loss Models

A. Parameter Definitions

We define ac resistance in terms of the loss in a winding P_w as

$$R_{ac} = P_w / I_{ac,rms}^2 \quad (1)$$

where $I_{ac,rms}$ is the rms current in the winding. R_{ac} therefore depends on the current waveform. Considering sinusoidal waveforms allows R_{ac} to be defined as a function of frequency. Because we use (1) and do not consider mutual resistance [14],

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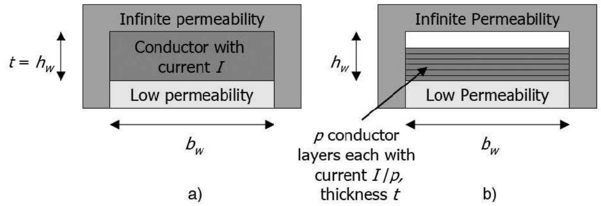

Fig. 1. Basic configurations analyzed. These are simplified models of the real situations of interest, as discussed in the text.

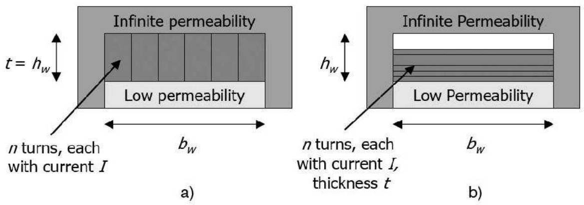

Fig. 2. A comparison of practical single-layer and multi-layer windings (a) Single-layer ($p = 1$) edge-wound foil with six turns ($N = 6$). (b) Barrel-wound foil with the same number of turns ($N = 6$), but with the number of layers equal to the number of turns ($p = N$). All turns are in series in both cases.

the analysis in this paper is limited to inductors and two-winding transformers with the same current waveform in both windings (scaled by the turns ratio).

B. Problem Definition

For the purpose of this analysis, the multi-layer and single-layer cases will be abstracted to be those illustrated in Fig. 1: a two-dimensional cross section of a single-layer distributed-gap inductor with a single thick turn of conductor, carrying current I, and the same core with a single-turn conductor divided into p layers of conductor each carrying current I/p. This simple configuration does not directly correspond to any practical situation, but it can represent many different practical situations, with varying degrees of approximation, as discussed below and in more detail in [10].

C. Application to Practical Winding Types

The configuration in Fig. 1 can be related to many different particular design situations [10]. One example is a comparison of a multi-turn barrel-wound foil winding and an edge-wound winding [15], as shown in Fig. 2 [10]. In this case, the number of layers in the barrel-wound foil, p, is equal to the number of turns, N. By breaking each layer of this design into multiple turns, it is possible to decouple the number of turns from the number of layers, as shown in Fig. 3 [10]. In order to ensure equal current in each conductor segment, this strategy can only be used to obtain a number of layers smaller than the number of turns, such that the conductors in different layers are connected in series. If the layers are in parallel, the currents will not be equal and the analysis does not apply, unless other measures are taken to ensure equal currents in the layers.

In wire windings, litz wire can be used to overcome the difficulties with paralleling conductors, if the strands of fine insulated

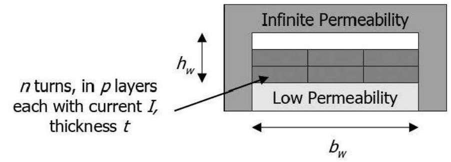

Fig. 3. Modified version of Fig. 2b with different numbers of turns and layers: six turns ($N = 6$) and two layers ($p = 2$) All turns are in series.

wire that constitute the litz conductor are twisted together in a "true litz" pattern such that each strand moves between positions in the bundle, to ensure equal currents in each strand. Litz wire windings do not have neatly structured layers at the strand level, but in most situations this does not significantly affect the loss [9], [16].

D. Application to Practical Inductors and Transformers

Fig. 1 shows an inductor with a distributed gap. Although a distributed gap can be a practical way to achieve a one-dimensional field, it is not very widely used in practice, and the practical cases of most interest are transformer windings and inductors with conventional lumped (discrete) air gaps. The loss analysis is the same for a simple transformer and the results of analyzing the configuration in Fig. 1 apply directly. For inductors, multiple small gaps can be used to approximate a distributed gap [17].

E. Models

For the situations shown in Fig. 1, the field is one-dimensional, and the Dowell model is an exact solution of Maxwell's equations. The ac resistance factor $F_r = \frac{R_{ac}}{R_{dc}}$ can be expressed as [18]

$$F_r = \Delta\left[\frac{\sinh 2\Delta + \sin 2\Delta}{\cosh 2\Delta - \cos 2\Delta} + \frac{2(p^2-1)}{3}\frac{\sinh\Delta - \sin\Delta}{\cosh\Delta + \cos\Delta}\right] \quad (2)$$

where Δ is the ratio of layer thickness to skin depth $\delta = \sqrt{\frac{\rho}{\pi\mu f}}$ where ρ is the conductor resistivity, μ is the conductor permeability, and f is the frequency of a sinusoidal current.

The validity of this model for the ideal situation shown in Fig. 1 is well established. It is also widely accepted and used for other geometries, such as layered round-wire windings. For a round-wire winding, it is no longer exact, and other models are superior [19], [11], [12]. However, our goal here is to understand general trends in order to select between types of windings, so we use (2) to explore different winding strategies. Once a particular strategy has been selected, other modeling can be used to more precisely predict performance.

For small values of Δ, as are typically advantageous in a multi-layer winding [6], (2) can be approximated as [20]

$$F_r = \left[1 + \left(\frac{5p^2-1}{45}\right)\Delta_{ml}^4\right] \quad (3)$$

where Δ_{ml} is the value of Δ for a multi-layer foil winding.

For large values of Δ, it is possible to calculate the loss based on the assumption that the current flows in a layer one skin-depth

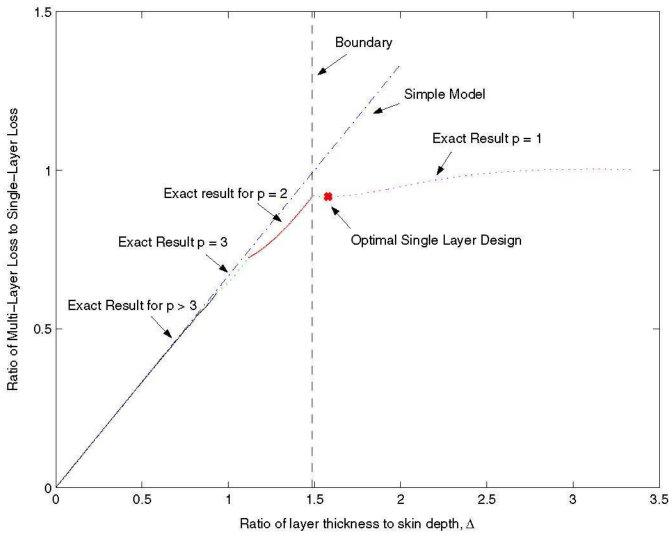

Fig. 4. Comparison of loss with a thick single-layer design to loss with an optimized multi-layer design, with the layer thickness constrained to the value on the horizontal axis. To the left of the vertical dotted line, multiple layers are used; to the right only a single layer. The large black dot is loss of a single layer design using the optimum thickness. The dash-dot line shows the simple model of (4), which works best for small Δ but works well for the full range where multi-layer designs are useful, up to $\Delta = 1.49$.

thick. We denote this resistance as R_δ, the resistance of a layer one skin depth thick.

III. Case 1: Minimum Thickness Constraint

In Section III of [10], the loss ratio between a multi-layer winding and a thick single-layer winding is shown to be

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\Delta_{min} \tag{4}$$

when layers of the multi-layer winding are constrained to a thickness $\Delta_{min}\delta$ and the optimal number of layers is used. For round wire (4) can be expressed in terms of the minimum wire diameter d_{min} as

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\left(\frac{3\pi}{16}\right)^{\frac{1}{4}}\frac{d_{min}}{\delta} = 0.584\left(\frac{d_{min}}{\delta}\right) \tag{5}$$

The ratios in (4) and (5) verify that the multi-layer winding design has lower loss than a single-layer winding design for small values of Δ_{min} and $\frac{d_{min}}{\delta}$. These equations also provide a quick and easy way to evaluate the improvement possible using a given technology that has a fixed minimum layer thickness.

In [10], the optimal number of layers (p_{opt}) is also found numerically using the full Dowell expression (2) to more accurately compare single-layer and multi-layer designs. Fig. 4 compares these exact results to the approximation given by (4), and confirms that (4) is accurate for $\Delta_{min} \leq 1.49$. The model error for $p \geq 5$ is less than 1%. The error is less than 1.8% for $p > 4$, less than 3.4% for $p > 2$, but increases to 8.6% over the range of designs for which the optimal value of p is 2.

The analysis in [10] leads to two conclusions in regards to a limited-thickness constraint. First, for a sinusoidal waveform and thickness-to-skin-depth ratio (using the thinnest practical layer), Δ_{min}, of less than 1.49, a multi-layer design is best. Second, single-layer loss can be reduced by a factor of $\frac{2}{3}\Delta_{min}$ using a multi-layer design with the optimal number of layers. Both of these conclusions are valid only when the optimal number of layers is used for these designs.

IV. Case 2: Maximum Number of Layers Constraint

With a fixed maximum number of layers, finding the lowest-loss multi-layer design requires finding the optimal thickness-to-skin-depth ratio, $\Delta_{ml_{opt}}$. To find this we minimize

$$\frac{F_r}{\Delta_{ml}} = \left[1 + \left(\frac{5p^2-1}{45}\right)\Delta_{ml}^4\right]\left(\frac{1}{\Delta_{ml}}\right) \tag{6}$$

which is proportional to loss because the dc resistance is inversely proportional to Δ_{ml}. Setting the derivative of (6) with respect to Δ_{ml} equal to zero results in:

$$\Delta_{ml_{opt}} = \left(\frac{15}{5p^2-1}\right)^{\frac{1}{4}} \tag{7}$$

To find ac resistance, and thus loss, for a multi-layer winding we first find F_r by substituting (7) into (3) resulting in

$$F_r = \frac{4}{3} \tag{8}$$

We then multiply by dc resistance, which can be expressed in terms of the resistance of a single-layer design, R_δ as

$$R_{dc,ml} = \frac{R_\delta}{p\Delta_{ml}} \tag{9}$$

to obtain

$$R_{ac,ml,opt} = \frac{4R_\delta}{3p\Delta_{ml_{opt}}} \tag{10}$$

The ratio of ac resistances for two designs is equal to the ratio of power losses, and so we can express the ratio of power loss in multi-layer and single-layer designs as

$$\frac{P_{ml,opt}}{P_{sl}} = \frac{4}{3p}\left(\frac{5p^2-1}{15}\right)^{\frac{1}{4}} \tag{11}$$

Assuming that a large number of layers is used in the multi-layer design, (11) can be simplified to

$$\frac{P_{ml,opt}}{P_{sl}} = \frac{1.013}{\sqrt{p}} \tag{12}$$

The ratio in (12) shows that the decrease in loss possible with an increase in the number of layers from a thick single-layer design is approximately equal to the inverse of $\sqrt{p}$. Doubling the number of layers results in a 28% decrease in loss while tripling the number of layers results in a 42% decrease in loss. Achieving these results requires using the optimal Δ_{ml} as given by (7), which, for large numbers of layers, can be simplified to

$$\Delta_{ml_{opt}} \approx \frac{1.3}{\sqrt{p}} \tag{13}$$

The analysis above was based on a simplified model. Fig. 5 is a comparison of (12) to designs that are numerically optimized using Dowell's analysis; it confirms that (12) is accurate to $< 1\%$ for $\Delta_{ml} \leq 0.47$. For $\Delta_{ml} < 1$, the error is less than 2%, but increases from 2% to 9.4% over the range of designs for which the optimal value of Δ_{ml} is 1 to 1.6. The variability around $p = 1$ results from the model assuming that Δ_{ml} is small, however for p from 1 to 3, $\Delta_{ml} \approx 1$.

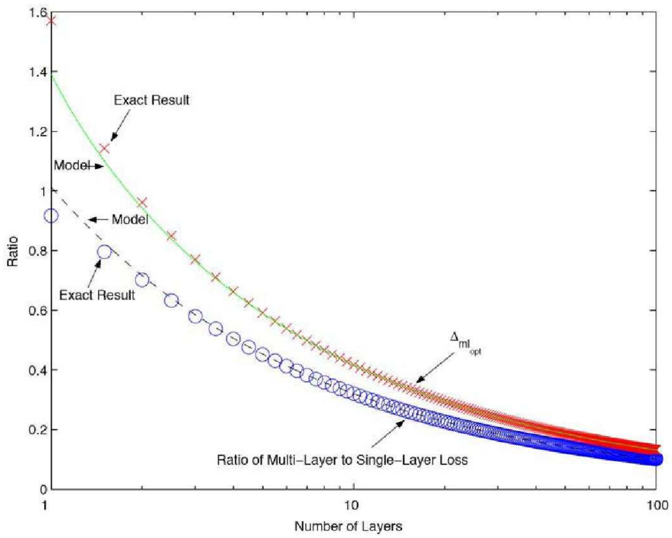

Fig. 5. Comparison of loss with a thick single-layer design to loss with an optimized multi-layer design, with the number of layers constrained to the value on the horizontal axis. The cross markers are for the exact model while the solid line is the simple model using (12). The optimal Δ_{ml} is also plotted versus number of layers. The circle markers are for the exact model solution while the dashed line is the simple model using (7).

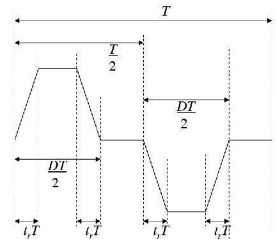

Fig. 6. Bipolar PWM waveform with a period T, rise time fraction t_r (and thus rise time t_rT), and duty cycle D.

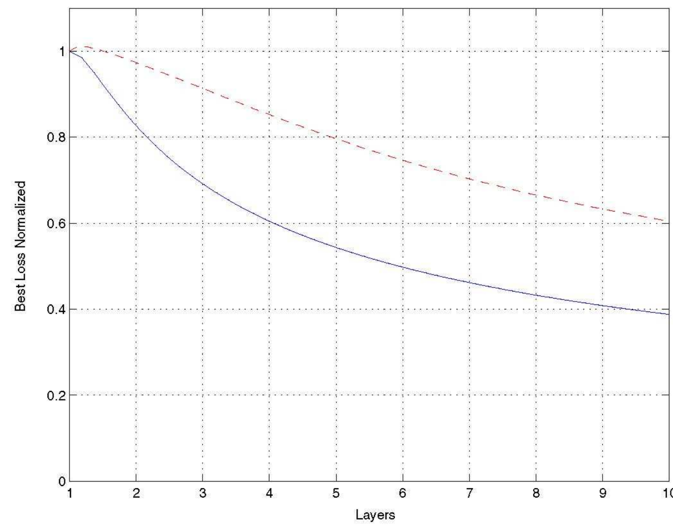

Fig. 7. Minimum loss for any given number of layers, normalized to the minimum single-layer loss, for two PWM waveforms. The solid line is for a bipolar PWM waveform as shown in Fig. 6 with a duty cycle of 50%, a rise time fraction of 10% (thus having moderate harmonic content). The dashed line is for the waveform with a duty cycle of 29%, a rise time fraction of 0.4%, (thus having high harmonic content).

V. Case 3: Effect of Harmonics

The ac resistance for a multi-layer design can be proportional to the square of frequency (3), whereas it is only proportional to the square root of frequency in a single-layer design. This can lead to a conclusion that single-layer designs are advantageous for waveforms with large harmonic content. This is often considered to be a disadvantage of litz wire, and some designers avoid litz wire for designs with large harmonic content.

To evaluate this hypothesis, we compared single-layer and multi-layer designs, using the same procedure in Section IV of [10]. The results reviewed below are similar to those in [10]. However, there are additional plots and previous errors have been corrected.

A. Bipolar PWM waveform

A bipolar PWM waveform with a pulse of width $\frac{DT}{2}$ and a rise time fraction of t_r, as shown in Fig. 6, was investigated first. The curves in Fig. 7 show the minimum loss for any given number of layers, normalized to the minimum single-layer loss. Note that the model (2) that was used allows computing loss for non-integer numbers of layers, even though ordinarily only integer and half-integer numbers of layers are physically meaningful.[1] We plot and discuss results for any non-integer value of the number of layers in order to show trends clearly, even though not all the results apply to practical configurations.

The solid-line curve in Fig. 7 is for the bipolar PWM waveform (Fig. 6) with $D = 50\%$ and $t_r = 1\%$. It shows that winding losses are lower using multi-layer designs. Numerical experiments showed that this is true for most practical parameter values for this waveform (Fig. 6). To determine whether this is always true, we searched for a counterexample. The dashed line curve in Fig. 7 is for a waveform with $D = 29\%$ and $t_r = 0.4\%$. This waveform has increasing loss for numbers of layers greater than one and peaks at 1.2 layers. However, by 1.5 layers a multi-layer design becomes superior to a single-layer design, and the multi-layer design is never much worse than a single-layer design—at the point where the single-layer design has the biggest advantage (1.2 layers) it is only 1% better than a multi-layer design.

Fig. 8 describes combinations of parameters t_r and D for which a single-layer design may be better than a multi-layer design. The number of layers needed for a multi-layer design to achieve lower loss than a single-layer design is listed on the contours of the plot. For the range shown on the plot (rise time fractions down

[1]The number of layers is counted from the zero MMF point (at the top next to the core in Fig. 1) to the maximum MMF point. Fractional layers result if the zero MMF point occurs within a layer.

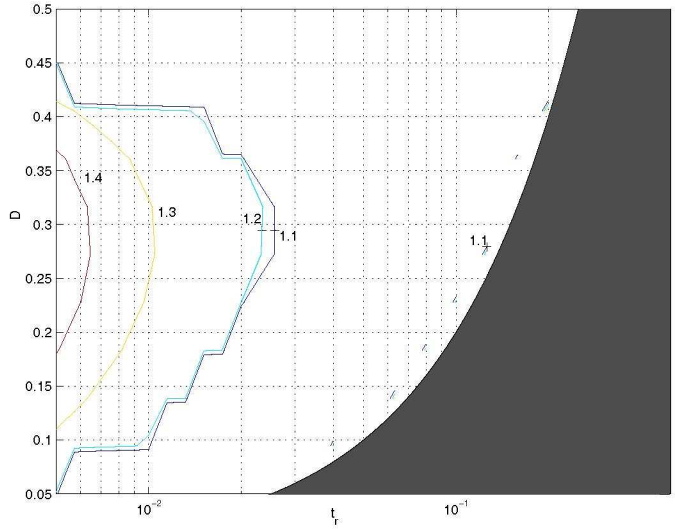

Fig. 8. Contour plot of number of layers necessary for a multi-layer design to have lower loss than a single-layer design for a bipolar PWM waveform as shown in Fig. 6 with the indicated rise time fraction (x-axis) and duty cycle (y-axis). The grey region designates invalid combinations of parameters ($t_r > \frac{D}{2}$).

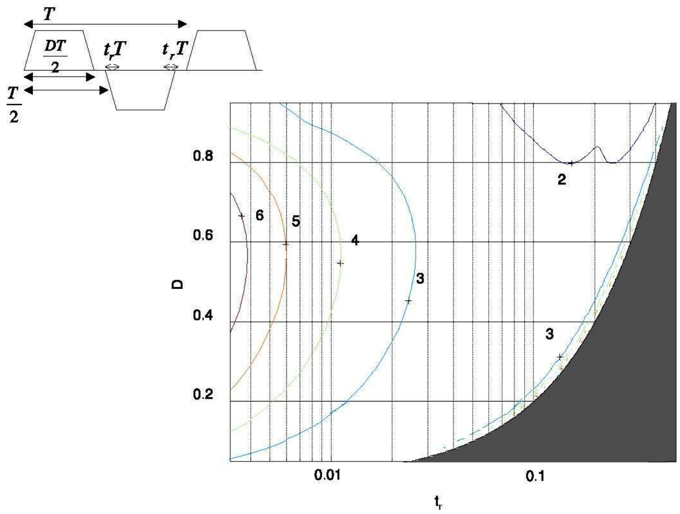

Fig. 9. Number of layers needed to achieve 20% reduced loss versus rise time fraction (x-axis) and duty cycle (y-axis) for a bipolar PWM waveform. The grey region designates invalid combinations of parameters ($t_r > \frac{D}{2}$).

to 0.5%), designs with 1.4 or more layers can always outperform single-layer designs. Thus, we can tentatively conclude that multi-layer designs are in fact generally superior to single-layer designs.

However, the improvement provided by a multi-layer design may not always be significant. The dashed-line curve in Fig. 7 shows a slight improvement at 3 layers, and by 5 layers a 20% improvement is possible. The cost of a large number of layers may be significant, and a significant improvement in loss would be needed to justify it. To investigate this issue, we arbitrarily choose 20% improvement as the threshold of significant loss improvement. The analysis of number of layers required to produce a 20% reduction in loss was repeated for all the common power electronics waveforms in [6] and those results are reviewed in the sections below.

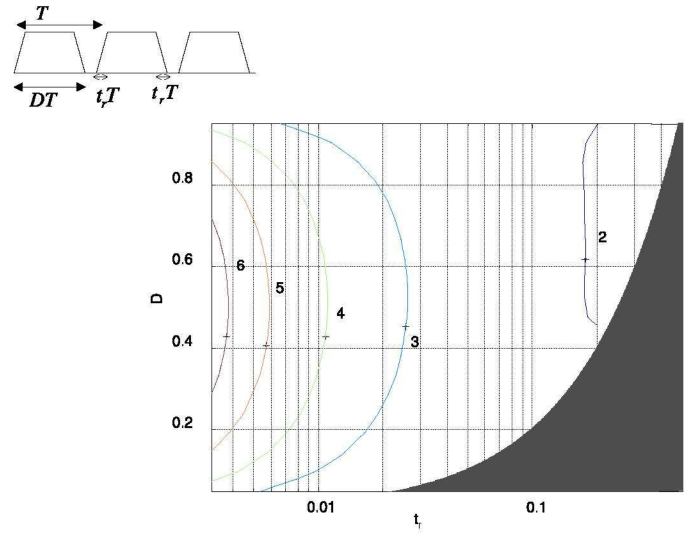

Fig. 10. Number of layers needed to achieve 20% reduced loss versus rise time fraction (x-axis) and duty cycle (y-axis) for the type of unipolar PWM waveform shown in the upper left corner. The grey region designates invalid combinations of parameters ($t_r > \frac{D}{2}$).

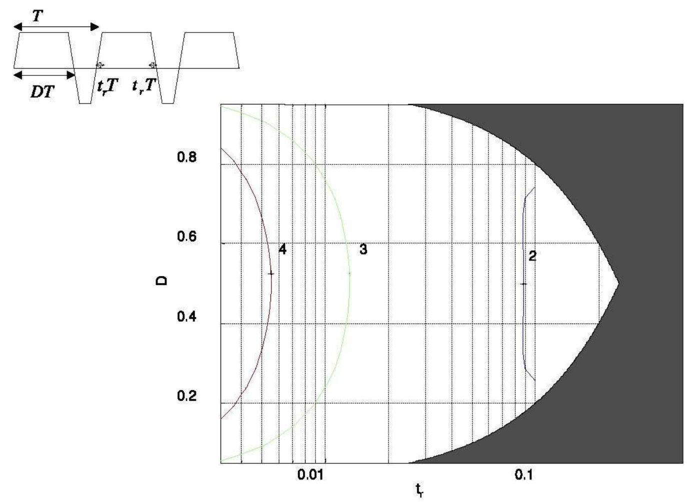

Fig. 11. Number of layers needed to achieve 20% reduced loss versus rise time fraction (x-axis) and duty cycle (y-axis) for the type of rectangular bipolar waveform plotted in the upper left corner. The grey region designates invalid combinations of parameters ($t_r > \frac{D}{2}$ and $t_r > \frac{1-D}{2}$).

B. Square Waveforms

The number of layers required to effect a 20% loss reduction for a bipolar PWM waveform is shown in Fig. 9. The number of layers required is highest for moderate duty cycles and small rise time fractions; about four layers for 1% rise time fraction and about five layers for 0.6% rise time fraction. The results for two other types of rectangular PWM waveforms are shown in Fig. 10 and Fig. 11, along with sketches of the waveforms. The results are very similar for all three waveforms, although the waveform in Fig. 11 requires slightly fewer layers.

C. Triangular and Sinusoidal Waveforms

A triangular waveform (such as the ac current in the filter inductor of a PWM dc-dc converter) typically requires two layers for a 20% reduction in loss as shown in Fig. 12. Results for the

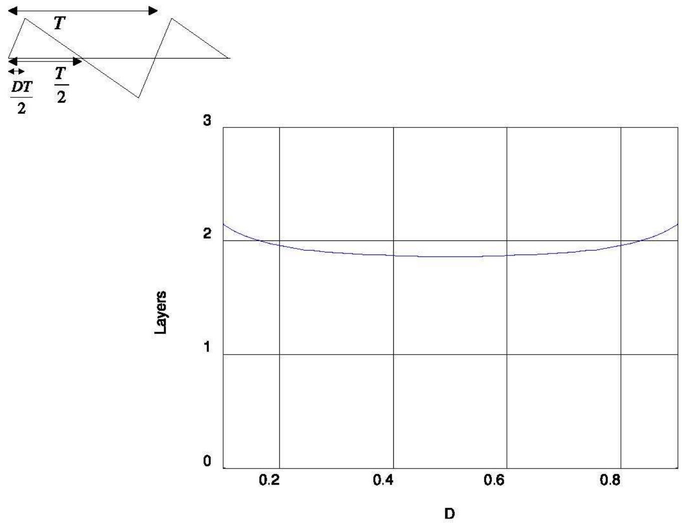

Fig. 12. Number of layers needed to achieve 20% reduced loss versus duty cycle for a simple triangular waveform.

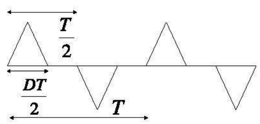

Fig. 13. Bipolar triangular pulse waveform with a period T and duty cycle D.

bipolar triangular pulse waveform shown in Fig. 13, the unipolar triangular pulse waveform shown in Fig. 14, the bipolar sinusoidal pulse waveform shown in Fig. 15, and the unipolar sinusoidal pulse waveform shown in Fig. 16 are shown in Fig. 17. For most duty cycles a minimum of two layers is needed to effect a 20% reduction in loss for these waveforms.

This analysis makes it clear that achieving substantial loss reduction for waveforms with strong harmonic content doesn't require many layers. For various square waveforms, those with moderate duty cycles and small rise time fractions are the waveforms that require the largest numbers of layers to achieve a significant reduction in loss with a multi-layer design. The minimum number of layers needed to get a 20% reduction in loss can be as high as six for a rise time fraction of 0.3%. Triangular and sinusoidal waveforms typically only need two or more layers to get 20% lower loss compared to a single-layer design.

VI. Summary

The work presented in Sections III–V investigated and quantified the situations when a single-layer design is optimal. This section summarizes useful results and equations from these sections.

A. Case 1: Constrained thickness

As discussed in Section III and [10], when the minimum layer thickness is limited, the improvement possible from a multi-layer design, for sinusoidal waveforms, can be expressed in terms of

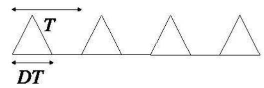

Fig. 14. Unipolar triangular pulse waveform with a period T and duty cycle D.

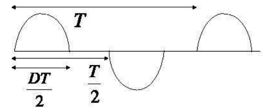

Fig. 15. Bipolar sinusoidal pulse waveform with a period T and duty cycle D.

the thickness-to-skin-depth ratio of the minimum thickness Δ_{min} as

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\Delta_{min}. \quad (14)$$

For round wire, this can be written in terms of the minimum wire diameter d_{min} as

$$\frac{P_{ml}}{P_{sl}} = 0.584\left(\frac{d_{min}}{\delta}\right) \quad (15)$$

The achievable thickness to skin depth ratio, Δ_{min}, needs to be less than 1.5 for multiple layers to offer improvement. The optimal number of layers,

$$p_{opt} \approx \frac{3}{\Delta_{min}^2} \quad (16)$$

must be used in order to achieve the loss reduction promised in (14) or (15).

B. Case 2: Constrained number of layers

For a sinusoidal waveform, the following equation can be used to determine the loss reduction possible through an increase in the number of layers.

$$\frac{P_{ml}}{P_{sl}} = \frac{1.013}{\sqrt{p}} \quad (17)$$

This calculation is accurate to less than 2% for $p \geq 2$ and $\Delta_{ml} \leq 1$. The optimal Δ_{ml} for this loss reduction is given by

$$\Delta_{ml_{opt}} = \left(\frac{15}{5p^2-1}\right)^{\frac{1}{4}} \approx \frac{1.3}{\sqrt{p}} \quad (18)$$

C. Case 3: High harmonic content

The following list summarizes the requirements for a 20% reduction in loss relative to a single-layer winding for the various waveforms examined in Section V.

- For triangular waveforms and triangular and sinusoidal pulse waveforms, both unipolar and bipolar, most reasonable parameters require two layers.
- For bipolar and unipolar PWM waveforms (Figs. 9 and 10), across most duty cycles, one needs:
  - three layers for $t_r \tilde{>} 2\%$
  - four layers for $1\% \tilde{<} t_r \tilde{<} 2\%$
  - five layers for $0.5\% \tilde{<} t_r \tilde{<} 1\%$
  - six layers for $0.3\% \tilde{<} t_r \tilde{<} 0.5\%$

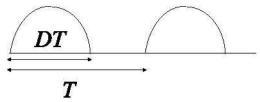

Fig. 16. Unipolar sinuosidal pulse waveform with a period T and duty cycle D.

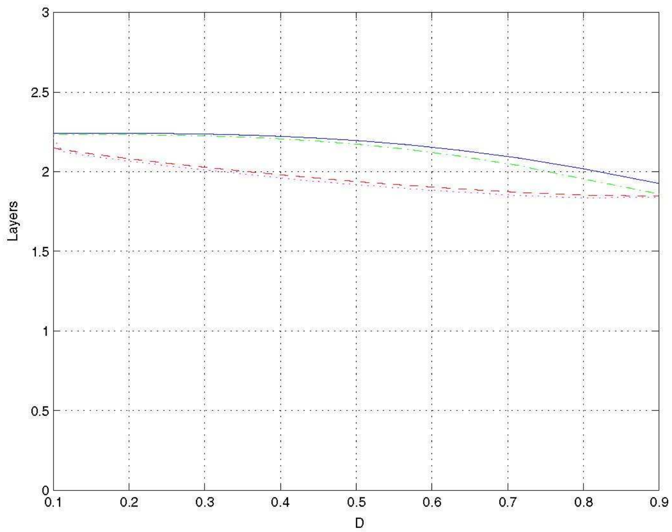

Fig. 17. Number of layers needed to achieve 20% reduced loss versus duty cycle for triangular and half-cycle sinusoidal pulse waveforms. The results for unipolar versions of these waveforms are the dashed and dotted curves. The results for bipolar versions are the solid and dot-dash curves.

- For rectangular bipolar waveforms (Fig. 11), across most duty cycles, one needs:
  - three layers for $t_r \tilde{>} 1\%$
  - four layers for $0.5\% \tilde{<} t_r \tilde{<} 1\%$
  - five layers for $0.3\% \tilde{<} t_r \tilde{<} 0.5\%$

VII. Conclusion

We can summarize our conclusions with two main points.

Firstly, we can conclude that, at least for practical power electronics waveforms without substantial dc or low-frequency components, a well designed multi-layer winding with a sufficient number of layers can have lower losses than a single-layer winding. This does not mean that any random multi-layer design is superior—it only holds if the winding is properly designed, taking into account all harmonics and optimizing the number of layers and/or the layer thickness.

Secondly, we conclude that waveforms with strong harmonic content do require more layers for a significant reduction in loss. However the number of layers required is often moderate, from two to six layers, with the larger numbers required for square current waveforms with fast rise times.

Regardless of which winding type gives lowest loss, the practical constraints in construction of the winding must also be considered. In many cases, the loss differences are not large, and the easiest type to construct should be selected. However, the differences are substantial in other cases. Useful formulas for evaluating the difference were summarized in Section VI. These formulas can provide general guidance on choosing between various approaches for winding design. Once an approach is chosen (or if several look promising), more accurate loss analysis and optimization approaches than those used here, along with evaluation of the manufacturability and cost, are recommended.

One important case that has not been addressed here is waveforms with substantial dc content. These situations will be analyzed in [13].

Acknowledgment

This work was supported in part by the United States Department of Energy under grant DE-FC36-01GO1106.

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